Point Process Model for Precisely Timed Spike Trains

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Supplementary Material

A conventional way of describing a point process is to describe the corresponding counting process which indicates how many points occurred within some interval. A counting process $\mathbf{N}(t)$ is a non-decreasing non-negative integer-valued random process, i.e. at each time t, $\mathbf{N}(t)$ is a discrete random variable and each sample path (realization of the process) is non-decreasing.

Definition 1 *The counting process* $\mathbf{N}(t)$ *of the* unreliable but precisely timed point process *can be described by two independent random variables* $\mathbf{R}$ *and* $\mathbf{T}$. $\mathbf{R}$ *is a Bernoulli random variable, i.e.* $\Pr[\mathbf{R} = 1] = p$ *and* $\Pr[\mathbf{R} = 0] = 1 - p$. $\mathbf{T}$ *is an arbitrary non-negative real-valued random variable with finite mean and corresponding distribution* $F(t)$. *The counting process is defined as,*

$$\mathbf{N}(t) = \mathbf{R} \cdot \mathbf{1}_{\mathbf{T} \leq t} \tag{1}$$

where $\mathbf{1}_{(\cdot)}$ *is the indicator function.*

$\mathbf{R}$ represents the *reliability*, if it takes the value 0, the corresponding event does not occur. $\mathbf{T}$ represents the actual occurrence pattern of the event which is the jitter distribution of the action potential. Note that $\mathbf{T}$ is completely ignored when $\mathbf{R}$ takes 0.

The definition 1 describes the full joint distribution for a finite collection of random variables $\{\mathbf{N}(A_i)\}$ for disjoint intervals $\{A_i\}$. Thus it defines a random process. Note that unlike Poisson or renewal process, the occurrences of action potential are highly correlated over time.

Definition 2 (Rate function) *If the derivative of* $F(t)$ *exists, we can define the instantaneous rate function as,*

$$\lambda(t) = \lim_{\Delta \to 0} \frac{Pr[N(t, t+\Delta) = 1]}{\Delta} = p\frac{dF(t)}{dt} \tag{2}$$

PTST consists of M indistinguishable and independent PTAPs with corresponding $\{p_i, F_i(t)\}_{k=1}^{M}$ and $\mathbf{N}_i(t) = \mathbf{R}_i \cdot \mathbf{1}_{\mathbf{T}_i \leq t}$.

$$N(t) = \sum_{i=1}^{M} N_i(t) \tag{3}$$

Note that the summation in (3) removes the origin of individual PTAP's. The probability density of realization therefore has to take all possible assignments from the spike train to the collection of PTAP's.

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To accommodate the additive action potentials and rate modulated responses, we also super-position a Poisson process.

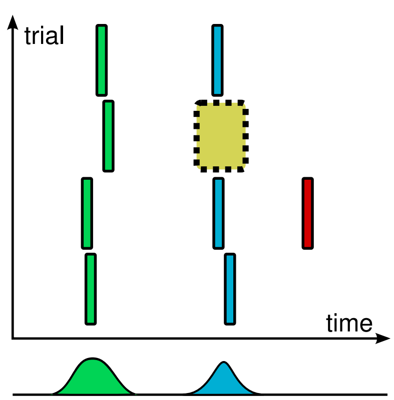
Definition 3 (Mixed precisely timed point process model)

$$N(t) = \sum_{i=1}^{M} N_i(t) + N_\nu(t) \tag{4}$$

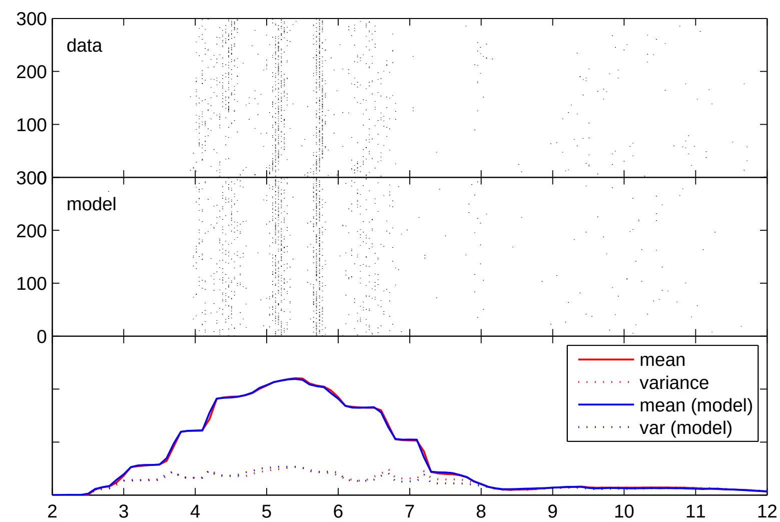
where $N_\nu(t)$ is a Poission process with intensity function $\nu(t)$.

Definition 4 (Rate function for PTST) *If the derivative of $F_i(t)$ exists for all i, we can define the instantaneous rate function of PTST as,*

$$\lambda(t) = \sum_{i=1}^{M} p_i \frac{dF_i(t)}{dt} + \nu(t) = \sum_{i=1}^{M} \lambda_i(t) + \nu(t) \tag{5}$$



(a) Illustration of differrent variabilities in a precisely timed spike train: precision, reliability, and addtive action potentials.



(b) Example spike trains (300 trials) obtained from *in vivo* and from model, and corresponding peri-stimulus time and variance histograms (3 ms sliding window).